

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

**Subject Name : Engineering Mathematics - I**

**Subject Code : 4TE01EMT3**

**Branch: B. Tech (All)**

**Semester : 1**

**Date : 14/03/2019**

**Time : 02:30 To 05:30**

**Marks : 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1                      Attempt the following questions:                      (14)**

- a)** nth derivative of  $y = e^{-x}$  is  
 (A)  $-e^{-x}$  (B)  $(-1)^n e^{-x}$  (C)  $(-1)^{n+1} e^{-x}$  (D) none of these
- b)** If  $y = \sin 2x \cos 2x$  then  $y_n$  equal to  
 (A)  $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$  (B)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$   
 (C)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$  (D) none of these
- c)** If  $y = \sin^{-1} x$ , then  $x$  equal to  
 (A)  $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$  (B)  $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$   
 (C)  $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \dots$  (D)  $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
- d)** The series  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$  represent expansion of  
 (A)  $\sin x$  (B)  $\cos x$  (C)  $\sinh x$  (D)  $\cosh x$
- e)**  $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} = \underline{\hspace{2cm}}$   
 (A) 2 (B)  $\log 2$  (C)  $\log 15$  (D)  $\log\left(\frac{5}{3}\right)$
- f)**  $\lim_{x \rightarrow \infty} x^n e^{-ax}$  ( $n$  being a positive integer and  $a > 0$ ) =  $\underline{\hspace{2cm}}$   
 (A)  $-1$  (B) 0 (C) 1 (D) None of these
- g)** If  $Q = r \cot \theta$ , then  $\frac{\partial Q}{\partial r}$  is equal to  
 (A)  $\cot \theta$  (B)  $-\cos^2 \theta$  (C)  $\cot \theta - r \operatorname{cosec}^2 \theta$  (D)  $\frac{1}{2} \cot \theta$



- h) If  $u = ax^2 + 2hxy + by^2$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to  
 (A)  $2u$  (B)  $u$  (C)  $0$  (D) none of these
- i) If  $f(x, y) = 0$ , then  $\frac{dy}{dx}$  is equal to  
 (A)  $\frac{\partial f / \partial x}{\partial f / \partial y}$  (B)  $\frac{\partial f / \partial y}{\partial f / \partial x}$  (C)  $-\frac{\partial f / \partial y}{\partial f / \partial x}$  (D)  $-\frac{\partial f / \partial x}{\partial f / \partial y}$
- j) If  $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$  is equal to  
 (A)  $1$  (B)  $-1$  (C)  $0$  (D) none of these
- k) If  $y = \cos \theta + i \sin \theta$ , then the value of  $y + \frac{1}{y}$  is  
 (A)  $2 \cos \theta$  (B)  $2 \sin \theta$  (C)  $2 \operatorname{cosec} \theta$  (D)  $2 \tan \theta$
- l) The number of solutions to the equation  $z^2 + \bar{z} = 0$  is  
 (A)  $1$  (B)  $2$  (C)  $3$  (D)  $4$
- m) If  $A$  is a non-zero column vector  $(n \times 1)$ , then the rank of matrix  $AA^T$  is  
 (A)  $0$  (B)  $1$  (C)  $n-1$  (D)  $n$
- n) The matrix  $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$  is given. The eigenvalues of  $4A^{-1} + 3A + 2I$  are  
 (A)  $6, 15$  (B)  $9, 12$  (C)  $9, 15$  (D)  $7, 15$

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

- a) If  $y = \frac{x}{x^2 + a^2}$  then find  $y_n$ . (5)
- b) Expand  $f(x) = \frac{e^x}{e^x + 1}$  in powers of  $x$  up to  $x^3$  by Maclaurin's series. (5)
- c) If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$  then show that (4)
- $$\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}.$$

**Q-3 Attempt all questions (14)**

- a) If  $y = \sin(m \sin^{-1} x)$  then prove that (5)
- $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$
- b) Prove that  $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$  (5)
- c) Evaluate:  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$  (4)

**Q-4 Attempt all questions (14)**



a) Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$  (5)

b) If  $u = \frac{y^2}{x}$ ,  $v = \frac{x^2}{y}$ , evaluate  $J = \left( \frac{x, y}{u, v} \right)$  and  $J' = \left( \frac{u, v}{x, y} \right)$  and hence (5)

verify that  $JJ' = 1$ .

c) Expand  $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$  in powers of  $(x-3)$ . (4)

**Q-5**

**Attempt all questions**

(14)

a) If  $u = \sec^{-1} \left( \frac{x^2 + y^2}{x - y} \right)$  then find  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ . (5)

b) Evaluate:  $\lim_{x \rightarrow 0} \frac{a}{x^2} \left[ \frac{\sin kx}{\sin lx} - \frac{k}{l} \right]$  (5)

c) Find  $n^{\text{th}}$  derivative of  $\tan^{-1} x$ . (4)

**Q-6**

**Attempt all questions**

(14)

a) Using the formula  $R = \frac{E}{I}$ , find the maximum error and percentage of (5)

error in R if  $I = 20$  with a possible error of 0.1 and  $E = 120$  with a possible error of 0.05 and  $R = 6$ .

b) Find the continued product of all the values of  $\left( \frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^{\frac{3}{4}}$ . (5)

c) Verify Caley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . (4)

**Q-7**

**Attempt all questions**

(14)

a) Find the inverse of  $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$  by Gauss-Jordan reduction (5)

method.

b) Find the fourth roots of unity and sketch them on the unit circle. (5)

c) If  $\tan(\alpha + i\beta) = x + iy$  then prove that  $x^2 + y^2 + 2x \cot 2\alpha = 1$ . (4)

**Q-8**

**Attempt all questions**

(14)

a) Investigate for what values of  $\lambda$  and  $\mu$  the equations (5)

$x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$ , have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

b) If  $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$  then prove that  $\lim_{n \rightarrow \infty} x_1 x_2 x_3 \dots x_n = -1$ . (5)

c) Check whether the following set of vectors is linearly dependent or linearly independent: (4)

$(1, 0, 1), (1, 1, 0), (1, -1, 1), (1, 2, -3)$

